Due Thursday, April 25, beginning of class

Assigned Problems

1. Suppose that 30% of the items in a large manufactured lot are of poor quality. Suppose also that a random sample of n items is to be taken from the lot, and let Q_n denote the proportion of items in the sample that are of poor quality. Find a value of n such that $P(0.2 \leq Q_n \leq 0.4) \geq 0.75$, using the Chebyshev inequality.

2. **(6.7.17)** Suppose that it is known that the number of items produced at a factory per week is a random variable X with mean 50.
   (a) What can we say about the probability that $X \geq 75$?
   (b) Suppose that the variance is 25. What can we say about $P(40 < X < 60)$?

3. How large a random sample must be taken from a given distribution in order for the probability to be at least 0.99 that the sample mean, $\bar{X}_n$, will be within 2 standard deviations from the mean of the distribution, μ?

4. Suppose that the random variables X and Y are iid, and X, $Y \sim N(0, 1)$.
   (a) The density of X/Y is given by . Verify that this is a probability density.
   (b) Determine $E(X/Y)$. Why does your answer make sense? (*Hint:* split the improper integral into two integrals.)

5. **(6.7.19)** Let $\bar{X}_n$ be the fraction of heads in 10,000 coin tosses. Consider $P(|\bar{X}_n - 1/2| \geq 0.01)$.
   (a) Use Chebyshev's inequality to generate bounds for this probability.
   (b) Use the normal approximation to estimate this probability.
   (c) How good were the bounds provided by Chebyshev's inequality?

6. **(6.7.21)** A person bets you that in 100 tosses of a fair coin the number of heads will differ from 50 by 4 or more. What is the probability that you will win this bet?

7. **(6.7.23)** Bill is a student at Cornell. In any given course he gets an A with probability 1/2 and a B with probability 1/2. Suppose the outcomes of his courses are independent. In his four years at Cornell he will take 33 courses. If he can get 22 A's and only 11 B's he can graduate with a 3.666 average. What is the probability that he will do this?

8. **(6.7.27)** A fair coin is tossed 2,500 times. Find a number m so that the chance that the number of heads is between $1,250 - m$ and $1,250 + m$ is approximately 2/3.

9. **(6.7.31)** A basketball player makes 80% of his free throws, on average. Use the normal approximation to compute the probability that in 25 attempts he will make at least 23.

10. In class, we stated the theorem that if $X \sim N(\mu, a)$ and $Y \sim N(\nu, b)$, then $X+Y \sim N(\mu+\nu, a+b)$. The proof of this result is given in your textbook. Write out this proof in your own words. Your explanation should clearly motivate each step in the proof. While the mathematical steps will be the same, you may feel the need to show additional steps or change the order, which is fine; *copying down the text without significant changes will not be sufficient for credit.*