Due Thursday, March 14, beginning of class

Assigned Problems

1. We can use computer software to simulate Markov chains, and we will do more this of next week. If you have not already, download Matlab by following the directions for computer software on the syllabus. As a warmup assignment for simulating Markov chains, use this transition matrix from class to answer the following questions.

$$p = \begin{bmatrix} & \textbf{Rainy} & \textbf{Nice} & \textbf{Snowy} \\ \textbf{Rainy} & 1/2 & 1/4 & 1/4 \\ \textbf{Nice} & 1/2 & 0 & 1/2 \\ \textbf{Snowy} & 1/4 & 1/4 & 1/2 \end{bmatrix}$$

   (a) Calculate p^{10}. *Hint:* Open up a new script and enter

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p = [0.5, 0.25 ,0.25 ; 0.5, 0, 0.5 ; 0.25, 0.25, 0.5];
p^(10)
```

   (b) Suppose you are locked in a room and don't know what the weather is outside, but you will escape in 10 days. If you assume that there is an equal chance of it being any type of day today, you can determine the chances of it being any of the types of day after 10 days by calculating [1/3, 1/3, 1/3]*p. What do you obtain?

2. **(4.7.1)** What values of x, y, and z will make the below matrices transition probabilities?

$$\text{(a)} \begin{bmatrix} 0.5 & 0.1 & \text{x} \\ \text{y} & 0.2 & 0.4 \\ 0.3 & \text{z} & 0.1 \end{bmatrix} \qquad \text{(b)} \begin{bmatrix} \text{x} & 0.1 & 0.7 \\ 0.2 & 0.3 & \text{y} \\ 0.6 & \text{z} & 0.2 \end{bmatrix}$$

3. **(4.7.4)** A person is flipping a coin repeatedly. Let X_n be the outcome of the two previous coin flips at time n; for example, the state might be HT to indicate that the last flip was T and the one before that was H.

   (a) Compute the transition probability p for the Markov chain.

   (b) Find p^2.

4. **(4.7.5)** A taxicab driver moves between the airport A and two hotels B and C according to the following rules. If he is at the airport, he will go to one of the two hotels next with equal probability. If at a hotel then he returns to the airport with probability 3/4 and goes to the other hotel with probability 1/4.

   (a) Find the transition matrix for this chain.

   (b) Suppose the driver begins at the airport at time $n = 0$. Find the probability for each of his three possible locations at time $n = 2$ and the probability that he is at hotel B at time $n = 3$.

5. **(4.7.8)** Market research suggests that in a 5-year period, 8% of people with cable television will get rid of it and 26% of those without it will sign up for it. Compare the predictions of the Markov chain model with the following data on the fraction of people with cable TV: 56.4% in 1990, 63.4% in 1995, and 68.0% in 2000.

6. **(4.7.39)** Two competing companies are trying to buy up all the farms in a certain area to build houses. In each year, 10% of farmers sell to Company 1, 20% sell to Company 2, and 70% keep farming. Neither company ever sells any of the farms that they own, so eventually all the farms will be sold to these companies. How many will be owned by Company 1?

7. **(4.7.41)** The AlphaBeta Company gives each of its employees the title of programmer (P) or project manager (M). In any given year, 70% of programmers remain in that position, 20% are promoted to project manager, and 10% are fired (state X). 95% of project managers remain in that position, while 5% are fired. You are trying to estimate whether or not you should apply for a posted programmer position. How long on average does a programmer work before they are fired?

8. **(4.7.42)** At a nationwide travel agency, newly hired employees are classified as beginners (B). Every 6 months the performance of each agent is reviewed. Past records indicate that transitions through the ranks to intermediate (I) and qualified (Q) are according to the following Markov chain where F indicates workers that were fired:

$$\begin{bmatrix} & \mathbf{B} & \mathbf{I} & \mathbf{Q} & \mathbf{F} \\ \mathbf{B} & 0.45 & 0.4 & 0 & 0.15 \\ \mathbf{I} & 0 & 0.6 & 0.3 & 0.1 \\ \mathbf{Q} & 0 & 0 & 1 & 0 \\ \mathbf{F} & 0 & 0 & 0 & 1 \end{bmatrix}$$

   (a) What fraction are eventually promoted?

   (b) What is the expected time until a beginner is fired or becomes qualified?

9. **(4.7.46)** Six children (Dick, Helen, Joni, Mark, Sam, and Tony) play catch. If Dick has the ball he is equally likely to throw it to Helen, Mark, Sam, and Tony. If Helen has the ball she is equally likely to throw it to Dick, Joni, Sam, and Tony. If Sam has the ball he is equally likely to through it to Dick, Helen, Mark, and Tony. If either Joni or Tony gets the ball, they keep throwing it to each other. If Mark gets the ball he runs away with it.

   (a) Find the transition probability.

   (b) Suppose Dick has the ball at the beginning of the game. What is the probability that Mark will end up with it?

News Assignment 5

(This is ***not*** the same description as last week. Updates to the assignment have been bolded.)

Find a news article, popular science piece, or scientific article that discusses or applies some aspect of probability theory **that we have learned in Chapters 2, 3, or 4**. This assignment is designed to provide structure for exploring the many ways in which probability is used in the world around us, and to help you prepare for finding a project topic of interest to you and your group members.

Prepare a typed, or neatly hand-written, summary of your chosen article. It should be between 1/2 and 1 page in length, and **to receive full credit**, it should contain the following components:

1. A citation for the article, in either MLA or APA format.
2. A 1-2 paragraph summary of the article contents, including the thesis and main argument. The idea here is for someone else in the class to be able to understand what the article was about without having to read it or have familiarity with subject matter that is not probability-related.
3. A 1 paragraph summary of how the article relates to a specific topic in our probability course. We don't need to have covered it yet, but it should either be a topic on the schedule, or an extension of a topic listed. If we haven't covered the probability topic yet, include a description of it in qualitative terms or in terminology that has been discussed in class.

Turn this summary in *separately* from the main homework assignment, since we will be discussing them in class.