Below are suggestions of projects. You also have the option to choose your own project, but please have it approved by the instructor if you do. After teams have been formed, please notify me with your choice of project (hillk@umn.edu), indicating a first and second choice in case another team has already chosen your first choice.

Investing Project

An investor has \$40,000 to invest. She is considering investments in savings, municipal bonds, and stocks, which have an average yearly return of 7%, 9%, and 14%, respectively. Because there are varying degrees of risk involved in the investments, the investor listed the following goals for her portfolio:

1. A yearly return of at least \$5,000.
2. An investment of at least \$10,000 in stocks.
3. The investment in stocks should not exceed the combined total in bonds and savings.
4. A liquid savings account between \$5,000 and \$15,000.
5. The total investment must not exceed \$40,000.

As we can see, the investor has more than one objective. Unfortunately, as is often the case with real-world problems, not all goals can be achieved simultaneously. What is the best possible investment strategy for the investor?

Forestry Project

A forester must decide when to cut trees which have been planted for timber. At time t, the forester cuts N trees. The selling price is \$5 per cubic feet of wood. The combined planting and harvesting costs are \$50,000 plus \$20 per tree. The volume v of wood (in hundreds of cubic freed) of the tree depends on time. It satisfies the following model of plant growth:

$$\frac{dv}{dt} = k_1 v - k_2 v^2 \quad v(0) = v_0.$$

Some possible values: $v_0 = 0.0001$, $k_1 = 1$, $k_2 = 0.1$, and t is in years.

Assuming the forester would like to plant more trees after the harvest, and harvest them at a time $2t$, $3t$, $4t$, etc., what would you recommend for the value of t? That is, consider several consecutive harvests.

Suppose you include inflation in your model, so that the costs and prices increase with a factor of e^{rt}, $r > 0$. Also, suppose you discount the final value of the trees (profits - costs) with a factor of $e^{-\mu t}$, $\mu > 0$, where μ is the nominal annual interest rate. This discount factor models the fact that the trees are an investment, so their "uncut" value is discounted by the amount of interest that could have been earned if they had been harvested and the money invested. Choose reasonable values of r and μ and show how it affects the optimal time of harvest.

Pesticide Project

A farmer raises a crop over 5 months. The farmer may choose to spray the crips with a pesticide once during those five months, in order to minimize the loss of revenue. The pesticide treatment is a one-time cost per crop cycle, which a fixed cost of \$500 plus the price per gallon of pesticide used. Crop losses due to pests are time dependent, and are proportional to the population size of the pest $p(t)$ at any time. The total amount of losses for the crop consists of all the loss due to pests over the five month growing period.

One model for the size of an insect population is

$$p(t) = p(0) + \frac{q}{r}e^{rt} - \frac{q}{r},$$

where r is the birth rate and q is a boost factor due to insect immigration to the area.

Data indicates that if the farmer sprays an amount of pesticide u, then it has a varied effect: if u is less than some critical amount, then there is no effect on the pest population, but if u is greater than this critical amount, the surviving insect population decreases as u increases. After the farmer sprays the pesticide, the pest population grows according to the equation above, where $p(0)$ is the reduced population.

Is it financially worthwhile for the grower to spray the pesticide? If so, when should it be sprayed, and how much?

Task Completion Project

Consider a machine which does 3 different tasks. The tasks are brought to the machine at a rate of one per minute, but the machine processes the different tasks at different rates. The machine can only perform one task at a time. While it is processing a task, the other tasks have to wait. For example if the machine is working on a task of type 1, then the number of tasks waiting for service changes in the following way: the number of tasks of type 2 and 3 both increases at a rate of one per minute, since they are waiting for service. But the number of type 1's is changing by a rate of one per minute minus the processing rate for type 1's.

While the tasks are waiting for processing, there is a cost in lost production time which depends on the number of jobs which are waiting. For example, if there are $n = 3$ possible tasks, and at time t the machine works on task 1 while $x_i(t)$ of type i waits, then the cost at time t is

$$\sum_{i=1}^{3} c_i x_i(t).$$

The coefficients c_i are constants, which are different for each task. Consider systems with different processing rates and different costs c_i for the waiting tasks. Can you find an optimal strategy for the order in which the tasks should be processed, in order to minimize the total costs over a finite interval of time? Were the costs or the processing times more important in your decision?

Personnel Scheduling Project

Nabla Air Lines is adding more flights to and from Minneapolis, and it needs to hire more check-in agents. However, it is not clear how many should be hired, considering the need for both cost control and providing a satisfactory level of customer service.

Based on the updated flight schedule, the airline has determined the *minimum* number of check-in agents that need to be on-duty at different times of the day to provide a satisfactory level of service. The table below indicates the minimum number of agents needed for the given time periods, along with the shift number and pay rate that time period corresponds to.

Time Period	**Time Periods Covered** Shift					**Minimum Number of Agents Needed**
	1	2	3	4	5	
6:00 AM to 8:00 AM	X					48
8:00 AM to 10:00 AM	X	X				79
10:00 AM to Noon	X	X				65
Noon to 2:00 PM	X	X	X			87
2:00 PM to 4:00 PM		X	X			64
4:00 PM to 6:00 PM			X	X		73
6:00 PM to 8:00 PM			X	X		82
8:00 PM to 10:00 PM				X		43
10:00 PM to Midnight				X	X	52
Midnight to 6:00 AM					X	15
Daily cost per agent	$170	$160	$175	$180	$195	

The entries regarding shift time reflect one of the provisions in the company's contract with the union that represents the check-in agents. The provision is that each agent work an 8-hour shift 5 days per week. The authorized shifts are

Shift 1: 6:00 AM to 2:00 PM
Shift 2: 8:00 AM to 4:00 PM
Shift 3: Noon to 8:00 PM
Shift 4: 4:00 PM to midnight
Shift 5: 10:00 PM to 6:00 AM

The X's in the main body of the table show the hours covered by the respective shifts. Because some shifts are less desirable than others, the wages specified in the contract differ by shift. The problem is to determine how many check-in agents should be assigned to the respective shifts each day to minimize the *total* personnel cost for agents (based on this bottom row), while meeting or exceeding the service requirements given in the rightmost column.

Newspaper Project

Reconsider Exercise 7 in Section 2.4, but do not assume linear relationships between increasing prices and the corresponding effects. Consider different nonlinear relationships between sales and ad expenditures, and determine how the assumptions affect the maximization of profit. Contrast the advantages and disadvantages of your models, keeping in mind issues such as computational complexity, sensitivity, and accurate and reasonable modeling. Are the results from using a linear assumption maintained?

Nutrition Project

A rancher has determined that the minimum weekly nutritional requirements for an average-sized horse include 40 lbs of protein, 20 lbs of carbohydrates, and 45 lbs of roughage. These are obtained from the following sources, in varying amounts, at the prices indicated below. Formulate a mathematical model to determine how to meet the minimum nutritional requirements at minimum cost.

	Protein (lb)	Carbohydrates (ld)	Roughage (lb)	Cost
Hay (per bale)	0.5	2.0	5.0	$1.80
Oats (per sack)	1.0	4.0	2.0	3.50
Feeding blocks (per block)	2.0	0.5	1.0	0.40
High-protein concentrate (per sack)	6.0	1.0	2.5	1.00

Air Pollution Project

In Steel City, uncontrolled air pollution from the local steel company's furnaces is ruining the appearance of the city and endangering the health of its residents. The shareholders revolted and elected a new board of directors, who will work to increase the air quality standards for the Steel City airshed.

The three main types of pollutants in this airshed are particulate matter, sulfur oxides, and hydrocarbons. The new standards require that the company reduce its annual emission of these pollutants by the amounts shown in the table below. Your task is to determine how to achieve these reductions in the most economical way.

Pollutant	**Required Reduction in Annual Emission Rate (Million Pounds)**
Particulates	60
Sulfur oxides	150
Hydrocarbons	125

The steelworks has two primary sources of pollution, namely, the blast furnaces for making pig iron and the open-hearth furnaces for changing iron into steel. In both cases the engineers have decided that the most effective types of abatement methods are (1) increasing the height of the smokestacks, (2) using filter devices (including gas traps) in the smokestacks, and (3) including cleaner, high-grade materials among the fuels for the furnaces. Each of these methods has a technological limit on how heavily it can be used (e.g., a maximum feasible increase in the height of the smokestacks), but there is also considerable flexibility for using the method at a fraction of its technological limit. The table below shows how much emissions (in millions of pounds per year) can be eliminated from each type of furnace by fully using any abatement method to its technological limit. For purposes of analysis, it is assumed that each method also can be used less fully to achieve any fraction of the emission-rate reductions shown in this table. Furthermore, the fractions can be different for blast furnaces and for open-hearth furnaces. For either type of furnace, the emission reduction achieved by each method is not substantially affected by whether the other methods also are used.

	Taller Smokestacks		Filters		Better Fuels	
Pollutant	Blast Furnaces	Open-Hearth Furnaces	Blast Furnaces	Open-Hearth Furnaces	Blast Furnaces	Open-Hearth Furnaces
Particulates	12	9	25	20	17	13
Sulfur oxides	35	42	18	31	56	49
Hydrocarbons	37	53	28	24	29	20

After these data were developed, it became clear that no single method by itself could achieve all the required reductions. On the other hand, combining all three methods at full capacity on both types of furnaces (which would be prohibitively expensive if the company's products are to remain competitively priced) is much more than adequate. Therefore, the engineers concluded that they would have to use some combination of the methods, perhaps with fractional capacities, based upon the relative costs. Furthermore, because of the differences between the blast and the open-hearth furnaces, the two types probably should not use the same combination.

An analysis was conducted to estimate the total annual cost that would be incurred by each abatement method. A method's annual cost includes increased operating and maintenance expenses as well as reduced revenue due to any loss in the efficiency of the production process caused by using the method. The other major cost is the start-up cost (the initial capital outlay) required to install the method. To make this one-time cost commensurable with the ongoing annual costs, the time value of money was used to calculate the annual expenditure (over the expected life of the method) that would be equivalent in value to this start-up cost. This analysis led to the total annual cost estimates (in millions of dollars) given in the table below for using the methods at their full abatement capacities. It also was determined that the cost of a method being used at a lower level is roughly proportional to the fraction of the abatement capacity given in the table above that is achieved. Thus, for any given fraction achieved, the total annual cost would be roughly that fraction of the corresponding quantity in the table below.

Abatement Method	Blast Furnaces	Open-Hearth Furnaces
Taller smokestacks	8	10
Filters	7	6
Better fuels	11	9

Miscellaneous Projects

Here is a listing of brief project topics, if none of the above appeals to you.

1. **Redfish industry.** In the 1980s, eating redfish became so popular that they were fished almost to the point of extinction. Now, their commercial harvest is banned in the U.S. to promote the growth of the redfish population. Formulate a proposed model for the sustainable harvesting of wild redfish. What is the optimum sustainable strategy to ensure the largest profit? Justify your label of "sustainable" or "unsustainable" commercial fishing schemes.

2. **Swinging on a swing set.** A child pumps his legs back and forth to increase height on a swing set. Create a model for this process, and answer the legendary question: "How high can you swing?"

3. **Composting.** Many people have backyard compost bins, but pay very little attention to what happens inside them. Model the composting process. Can you recommend an optimal food mixture for a rich compost result, or a method for creating useable compost in the shortest amount of time?