Due September 10, beginning of class

Instructions: Show your work. An explicit, logical, and neat presentation of each solution is required, with explanations written in complete sentences. Numbered problems are from the text.

1. Given the differential equation
$$\frac{dy}{dx} = y - x + 1$$
   (a) Determine the solution to this DE with initial condition $y(-3) = 1$.
   (b) Determine the solution to this DE with initial condition $y(-3) = -3$.
   (c) Use Matlab and the code on pp.12-13 in the textbook to create a direction field for this DE. Sketch or provide a printout of this direction field, along with the solutions to the IVPs found in parts (a) and (b).
   (d) Approximate the value at $x = 1$ of the solutions to the DE with initial conditions $y(-3) = 0.99$ and $y(-3) = 1.01$. Then approximate the value at $x = 1$ of the solutions to the DE with initial conditions $y(-3) = -2.99$ and $y(-3) = -3.01$. What does this suggest about the conditioning of the IVPs in parts (a) and (b)?
   (e) Determine the conditioning of these IVPs using the methods of Section 1.2.
2. Chapter 1, Problem 5
3. Chapter 1, Problem 6
4. Consider the Duffing oscillator,
$$x'' + \delta x' + \beta x + \alpha x^3 = \gamma \cos(t).$$
   This is an example of a simple DE that can exhibit chaos. For now, let's simulate it by avoiding parameter values that lead to chaos.
   (a) To approximate the solution numerically, we need to transform this 2nd order DE into a system of 1st order DEs. Use the substitution $x' = y$ to rewrite this as a system of 1st order DEs. Write your answer in *standard form*, i.e.
$$\begin{aligned} x' &= \cdots \\ y' &= \cdots \end{aligned}$$
   (b) Modify the file '`VanderPol.m`' to approximate the solution of the Duffing oscillator when $\delta = 0.1$, $\beta = 0.72$, $\alpha = 0.13$, and $\gamma = 2$. Plot the numerical solution for several different initial conditions between $x(0) = 0$ and $x(0) = 3$ and for $x'(0) \equiv y(0) = 2$ and compare their behavior. What happens to the solution $x(t)$ as $t \to \infty$? Show a plot that supports your conclusion.
5. Consider the model chemical reaction,
$$A + X \underset{k_{-1}}{\overset{k_1}{\rightleftharpoons}} 2X,$$
   in which one molecule of X combines with one molecule of A to form two molecules of X. This means that X stimulates its own production, a process called *autocatalysis*. This positive

feedback process leads to a chain reaction, which is eventually limited by a "back reaction" in which $2X$ returns to $A + X$. According to the law of mass action of chemical kinetics, the rate of an elementary reaction is proportional to the product of the concentrations of the reactants. We denote the concentrations by lowercase letters $x = [X]$ and $a = [A]$. Assume that there is an enormous surplus of chemical A, so that its concentration a can be regarded as constant. Then the equation for the kinetics of x is

$$\frac{dx}{dt} = k_1 a x - k_{-1} x^2,$$

where k_1 and k_{-1} are positive parameters called rate constants.

(a) Solve this DE with initial condition $x(0) = 2$ and parameter values $a = 4$, $k_1 = 1$, and $k_{-1} = 4$. Is this well-conditioned?

(b) Use Matlab to create a direction field for this DE. Sketch or provide a printout of this direction field, along with the solution to the IVP in part (a).

(c) Modify your code from problem 4 to create an accurate numerical approximation of the solution to this IVP using `ode45`. How does the numerical approximation to the solution compare to the analytical solution?