Small Fish in a Big Pond

There are many different ways to modify the logistic model. An example of one differential equation is below

$$\frac{dP}{dt} = kP\left(1 - \frac{P}{N}\right) - h,$$

which differs from the original by only in the "$-h$" at the end. We will define k as the *relative growth rate*, N as the *carrying capacity*, and h is the *harvesting constant*. The harvesting constant represents the number of individuals that are removed from the population per time unit.

Conservation Officer O'Hara[1] at the Department of Natural Resources is planning how to manage the walleye population in Mille Lacs Lake. Officer O'Hara must decide how many fish anglers will be allowed to catch each year and how many fish should he initially stock the lake with. From previous (exhaustive) research,[2] we know that the pond can support 36,000 walleye. With population measured in thousands, the relative growth rate is $k = 2$ and we have the differential equation

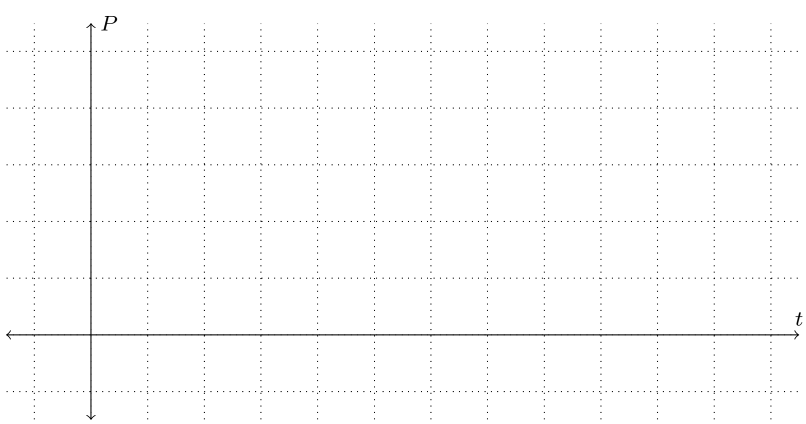
$$\frac{dP}{dt} = 2P\left(1 - \frac{P}{36}\right) - h.$$

Answer the following questions based on different harvesting constants.

1. Suppose no fish are allowed to be caught ($h = 0$) in the lake.

   (a) What are the equilibrium solutions to this differential equation?

   (b) Roughly sketch the direction field of the differential equation.



   (c) Describe the long-term behavior of the population for different initial values.

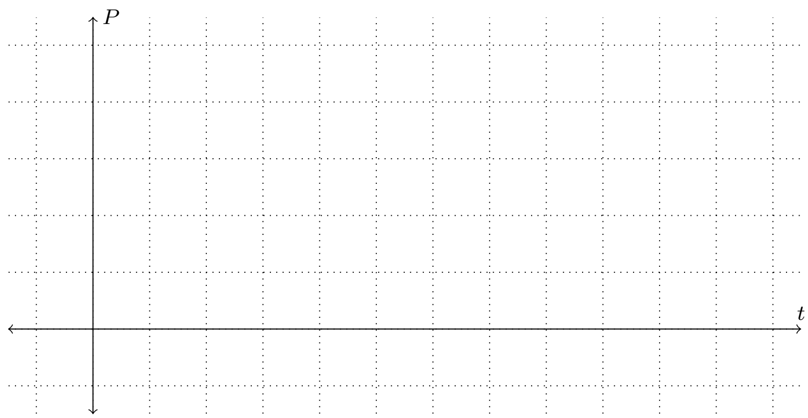
[1] Originally written by Eoin O'Hara (ohara003@umn.edu). Modified to incorporate Geogebra.

[2] These numbers were made up for the salmon population in a fictitious pond, but we changed the problem to refer to Mille Lacs because this is a current ongoing issue with the walleye population in the lake.

2. Suppose $10,000$ fish are allowed to be caught ($h = 10$) in the lake per year.

   (a) What are the equilibrium solutions to this differential equation?

   (b) Now that you have had some practice finding equilibrium solutions to this problem analytically, let's explore the problem numerically. Go to the course Moodle page and open the Geogebra file for Section 7.5. The applet shows the numerical solution to our model. Does Geogebra show similar results to what you got for the first question? Roughly sketch the direction field shown in Geogebra.

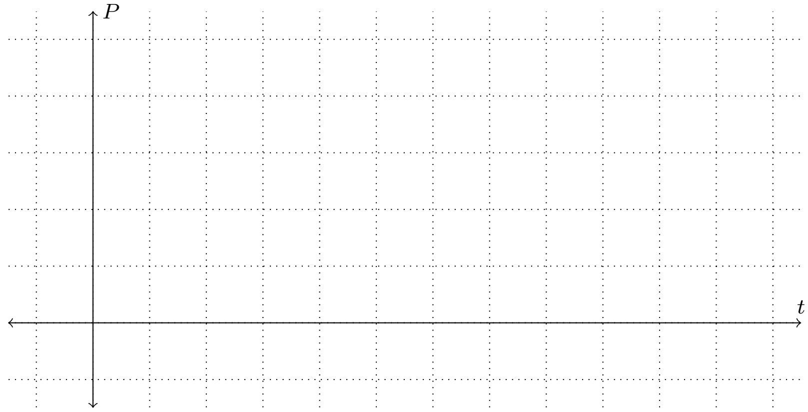


   (c) Describe the long-term behavior of the population for different initial values. In particular, what would happen if Officer O'Hara stocks $36,000$ fish initially?

3. Suppose $18,000$ fish are allowed to be caught in the lake per year. Set $h = 18$ in Geogebra, and use your observations to answer the following.

   (a) What are the equilibrium solutions to this differential equation?

(b) Sketch the direction field shown in Geogebra.

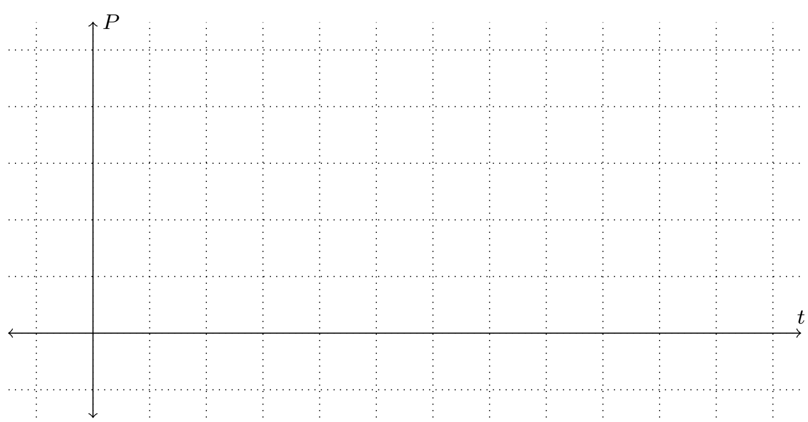


(c) Describe the long-term behavior of the population for different initial values.

4. Suppose 20, 000 fish are allowed to be caught ($h = 20$) in the lake per year.

(a) What are the equilibrium solutions to this differential equation?

(b) Sketch the direction field shown in Geogebra.



(c) Describe the long-term behavior of the population for different initial values.

5. Find the equilibrium solutions to the differential equation $\frac{dP}{dt} = 2P\left(1 - \frac{P}{36}\right) - h$ for a generic harvesting constant h.

6. (a) For a generic harvesting constant h, if the population is decreasing, will the population always die out? Explain.

(b) What would you recommend to Officer O'Hara? Be sure to include how many fish would you allow to be caught and how many fish would you initially stock. *You want a sustainable model.*