



4. Prove

$$\binom{2n}{n} = \sum_{j=0}^n \binom{n}{j}^2.$$

(Hint: Consider an urn with  $n$  red balls and  $n$  blue balls. Show that each side of the equation counts the number of ways to choose  $n$  balls from the urn.)

Real World Example: The court case *De Martini v. Power* dealt with a case of ballot irregularities in a small election. De Martini received 2,656 people votes and Power received 2954 votes. Later, 136 of the 5250 votes cast were declared invalid due to irregularities. Power wanted to overturn the election results, as his loss was only by 62 votes. However, in order to reverse the outcome of the election, at least 99 of the 136 fraudulent votes must have been cast for De Martini. Modeling this as an urn problem, we can see that

$$P(m \text{ of the irregular votes were counted for De Martini}) = \frac{\binom{2656}{m} \binom{2594}{136-m}}{\binom{5250}{136}}.$$

Summing from  $m = 99$  to 136, we find the probability that at least 99 of the irregular votes were for De Martini is  $7.492 \times 10^{-8}$ . Based on this, the court decided that the election result would stand.

For more information:

- Example 2.40 from your textbook
- Anita M. De Martini v. James M. Power, in the Court of Appeals of the State of New York (you can read the decision at [law.justia.com](http://law.justia.com))